ON THE MAGNETO-ROTON GAP AND THE LAUGHLIN STATE STABILITY

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Abstract.

It is argued that the Girvin, MacDonald and Platzman (GMP) evaluation of the magneto-roton spectrum, in spite of probably being a sensible estimate of the excitation spectrum around the real FQHE ground state, is not implying the variational stability of the Laughlin state. The suplementary corrections needed to produce a variational calculation around the $\nu = 1/3$ Laughlin state are evaluated approximately. The results indicate within the considered aproximation, the existence of lower lying states for a range of wavevector values.

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1 Introduction.

In a previous letter [1] the violation of a stability condition by the Single Mode Approximation for the evaluation of the excitation gap in FQHE ground state was argued. In other related works, the calculations of the collective mode dispersion, first for composite fermions in [2] and after from a Behe-Salpeter approach based in a phenomenological ansatz for the electron progrator [3], gave similar indications for the existence of lower energy states with broken translational invariance.

It should be expressed that attempts to give foundation for a broken translational symmetry in the FQHE ground state have been existing since the times of the discovery of the effect. Some of them can be traced out in Refs. [5]-[12]. Concretely, we have been working in a particular direction of thinking which based in the obtention of exact Hartree-Fock solution at fractional filling factors [13]-[17]. The existence and interesting properties of these solutions have been the main motivation for the expectation of a broken symmetry ground state.

In the present letter, the GMP evaluation of the magneto-roton spectrum is conceptually examined [18]. The aim is to precise its implications in connection with the variational stability of the Laughlin state. It is concluded that the GMP evaluation, while presumably being a valuable estimate of the real excitation spectrum around the true ground state, is not implying that magneto-roton states have greater energies than the Laughlin ones. Complementary terms should be also calculated in order to verify the Laughlin state variational stability. Their evaluation under the use of the approximate reduced density matrices given in [19], gives results signaling the existence of magneto-roton wave functions having lower energies than the Laughlin state at $\nu = 1/3$.

To start, let us consider the difference between the magneto-roton and Laughlin state energies

$$\epsilon(k) = <\phi_k|H|\phi_k> - <\phi_L|H|\phi_L> \tag{1}$$

where the magneto-roton state is given as usual by

$$|\phi_k >= \frac{1}{\sqrt{N}} \rho_k |\phi_L > \qquad (2)$$

and its norm is given by the projected static structure factor

$$s(k) =< \phi_k|\phi_k >= \frac{1}{N} < \phi_L|\rho_k^+ \rho_k|\phi_L > \qquad (3)$$

where N is the number of particles.

Here, all the conventions for the definitions of the density operators, Hamiltonian, etc. are the ones given in Ref. [20]. More specifically $z = x+iy$ is the complex representation of a 2D-position vector and $k = k_x + ik_y$ (or $q = q_x + iq_y$) the same representation for wavevectors. The magnetic field is taken along the negative z axis and the magnetic length is set equal to one. The symmetric gauge is also assumed. The projected electron Hamiltonian is given by

$$H = \frac{1}{2} \int \frac{d^2 q}{(2\pi)^2} \upsilon(\vec{q}) (\rho_q^+ \rho_q - \rho e^{-qq^*/2}) \qquad (4)$$

with ρ being the density of the state and the Coulomb potential $\upsilon(\vec{q}) = 2\pi/|\vec{q}|$. Finally, the projected density operator is given by

$$\rho_k = \sum_{j=1}^{N} exp\left[-ik\frac{\partial}{\partial z_j}\right] exp\left[-\frac{ik^*}{2} z_j\right]. \qquad (5)$$

After some algebraical transformations (1) can be written in form

$$\begin{aligned} \epsilon(k) &= \frac{< \phi_L|\rho_k^+ [H, \rho_k]|\phi_L >}{< \phi_L|\rho_k^+ \rho_k|\phi_L >} \\ &+ \frac{< \phi_L|\rho_k^+ \rho_k H|\phi_L > - < \phi_L|\rho_k^+ \rho_k|\phi_L >< \phi_L|H|\phi_L >}{< \phi_L|\rho_k^+ \rho_k|\phi_L >} \qquad (6) \end{aligned}$$

Then, the reality of $\epsilon(k) = \epsilon^*(k)$ allows to write for $\epsilon(k) = \frac{1}{2}(\epsilon(k) + \epsilon^*(k))$

$$\epsilon(k) = \Delta(k) + \delta(k), \tag{7}$$

where

$$\Delta(k) = \frac{< \phi_L|[\rho_k^+, [H, \rho_k]]|\phi_L >}{2 < \phi_L|\rho_k^+ \rho_k|\phi_k >}, \tag{8}$$

$$\delta(k) = \frac{\frac{1}{2} < \phi_L|H\rho_k^+\rho_k + \rho_k^+\rho_k H|\phi_L > - < \phi_L|\rho_k^+\rho_k|\phi_L >< \phi_L|H|\phi_L >}{< \phi_L|\rho_k^+\rho_k|\phi_L >}. \tag{9}$$

From relation (9) it follows that the formula used in Ref. [20] for the evaluation of $\Delta(k)$, the SMA mode energy, describe the excitation over the Laughlin state $|\phi_L >$ whenever $|\phi_L >$ is an exact eigenstate of H. In such a case the expression (9) for $\delta(k)$, vanish identically. Therefore, in order to conclude that the evaluation of $\Delta(k)$ in Ref. [20] implies the variational stability of the Laughlin state it should be verified that $\delta(k)$ can be disregarded. This simple conclusion is the main point of this letter.

Here a first consideration of the above question is presented. For this purpose an evaluation of (9) was done by making use of the reduced density matrices given in Ref. [19]

$$\rho_M(z', z) = \Big(\frac{\nu}{2\pi}\Big)^M \prod_{k=1}^{M} \Big\{exp\Big(- \frac{z_k z_k^*}{4} - \frac{z_k' z_k^{*\prime}}{4} + \frac{z_k z'^*}{2}\Big)\Big\}$$
$$\prod_{\substack{i,j=1 \\ i<j}}^{M} g((z_i' - z_j')^*(z_i - z_j)) \;\; M = 1, 2, 3, 4; \tag{10}$$

where $z = (z_1, \dots z_M), z' = (z_1', \dots z_M')$ and $g(r^2)$ is the pair correlation function of the Laughlin state. It should be stressed that only for $M = 1, 2$ the density matrices in (10) are almost exactly known. For $M = 3, 4$ the expresions (10) are approximate ones obtained under the assumption that three and four point density matrices are expressed as products of two-point correlation functions g. The use of the density matrices (10) allows to calculate the following expression for the correction $\delta(k)$

$$
\begin{aligned}
\delta(k) \;=\;& \frac{1}{2s(k)}(\frac{\nu}{2\pi})\int \frac{d^2q}{(2\pi)^2}v(\vec{q})(\exp(k^*q)+\exp(kq^*))\cdot\int d^2x\ \exp(i(\vec{k}+\vec{q})\vec{x})g(\vec{x}_1^2) \\
&+\frac{1}{s(k)}\Big(\frac{\nu}{2\pi}\Big)^2\int \frac{d^2q}{(2\pi)^2}v(\vec{q})\Big(\exp(k^*q/2)+\exp(kq^*/2)\Big)\cdot \qquad (11)\\
&\int d^2x_1d^2x_2\exp\Big(i\vec{k}\cdot\vec{x}_1+i\vec{q}\cdot\vec{x}_2\Big)\,g(\vec{x}_1^2)g(\vec{x}_2^2)g((\vec{x}_1-\vec{x}_2)^2) \\
&+\frac{1}{2s(k)}\cdot\Big(\frac{\nu}{2\pi}\Big)^3\int\frac{d^2q}{(2\pi)^2}v(\vec{q})\int d^2x_1d^2x_2d^2x_3\exp(i\vec{k}\cdot\vec{x}_1+i\vec{q}\cdot\vec{x}_2) \\
&g(\vec{x}_1^2)g(\vec{x}_2^2)[g(\vec{x}_3^2)g((\vec{x}_3-\vec{x}_1)^2)g((\vec{x}_3-\vec{x}_2)^2)g((\vec{x}_3-\vec{x}_2-\vec{x}_1)^2)-1]
\end{aligned}
$$

where $g(\vec{x}^2)$ is given by the acurrate analytical expression derived by Girvin

$$
g(\vec{x}^2)=1-\exp(-\vec{x}^2/2)+\sum_{n=0}^{\infty}\frac{2}{(2n+1)}\Big(\frac{\vec{x}^2}{4}\Big)^{2n+1}C_{2n+1}\exp\Big(-\frac{\vec{x}^2}{4}\Big) \qquad (12)
$$

in which the coefficient C_{2n+1} are reported in [10] for the 13 first integers and the values $\nu = 1/3$ and $\nu = 1/5$ for the filling factor.

The further evaluation of $\delta(k)$ through (11) was considered for the $\nu = 1/3$ state. In performing it, the Fourier transform of the Coulomb potential was regularized at long distances (small $\vec{k}$) in order that its zero momentum component vanish, that is $v(\vec{k}=0)=0$. This procedure allows to regularize and cancel singular terms associated to the non-decaying values of the pair correlation functions at infinity. The resulting finite terms are continuos upon the removal of the regularization. The numerical evaluation was performed approximatelly using the Monte Carlo algarithm implemented in Mathemathica 3.0. The number of sample points was incremented up to a stabilization of the calculated values was noticed. The results for the $\epsilon(k)=\Delta(k)+\delta(k)$ are shown in Fig. 1 by the continuos curve. The points correspond to the magneto-roton spectrun $\Delta(k)$.

The results in Fig.1 indicate, within the considered approximation, that the magneto-roton states could have lower energies than the Laughlin states. This occurs for wavevector values in excess of $kr_0 \sim 1.5$. At lower values of k the energy difference $\epsilon(k)$ tends to grow. This bahavior is similar to the

one obtained in the previous work [2] for the composite fermion excitation spectrum. In that case the growth reflected the tendency of the spectrum to reproduce the cyclotron resonances excitation at low wavevectors. A similar picture was also obtained in the work [3].

We want to stress that the instability of Laughlin state suggested by the present calculation does not invalidate the $\Delta(k)$ spectrum as an accurrate approximation for the exact collective mode. This statement is supported by the fact that the Laughlin state is undoubtely a good approximation for the exact ground state, then, the evaluation of the formula (8) for $\Delta(k)$ corresponding to the exact increase in energy over the true ground state could effectively furnish good results for the correct gap. Therefore, the present argue in this letter should not be interpreted as claiming the invalidation of the SMA approximation. The central point here supported is that the real ground state could be a weakly inhomogeneous state not very much differing from the Laughlin wavefunction.

In summary, it is underlined that the magneto-roton spectrum evaluation is not implying the variational the stability of the Laughlin state at $\nu = 1/3$. The additional terms needed in checking such stability are approximately calculated. The results indicate the existence of magneto-roton states lowering the energy of the $\nu = 1/3$ Laughlin wavefunction. The work in performing precise evaluations needed to confirm the existence of such states is being considered.

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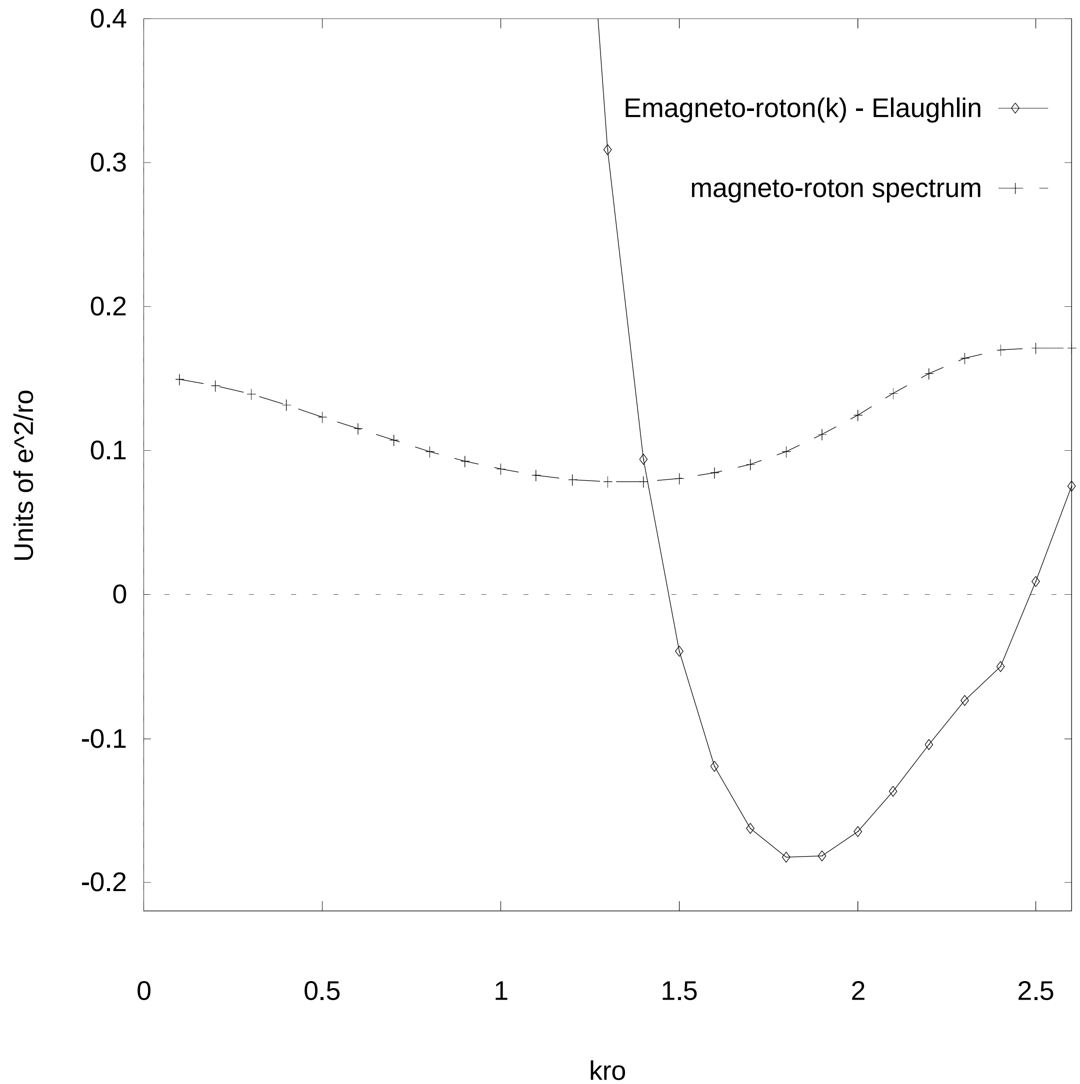
Emagneto-roton(k) - Elaughlin
magneto-roton spectrum
Units of e^2/ro
kro
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
0.5
1
1.5
2
2.5